

Chirality-preserving neutrino oscillations in an external magnetic field

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Abstract

Neutrinos propagating in matter acquire an effective electromagnetic vertex induced by their weak interactions with the charged particles in the background. In the presence of an external magnetic field the induced vertex affects the flavor transformations of mixed neutrinos in a way that, in contrast to the oscillations driven by an intrinsic magnetic moment interaction, preserve chirality. We derive the evolution equation for this case and discuss some of the physical consequences in environments such as a supernova. For small values of the square mass difference the resonance for neutrinos and antineutrinos occur within regions which are close. In that case, the resonance condition becomes independent of the vacuum parameters and is approximately the same for both.

As is now well known, under certain favorable conditions, large transformations from one neutrino flavor into another may take place in a medium, even for small neutrino mixing in vacuum[1]. The matter effects on the ν oscillations are taken into account by means of a potential energy or an index of refraction for each neutrino flavor, which can be calculated from the background contributions to the neutrino self-energy[2, 3, 4]. In addition to the energy-momentum relation of the neutrinos, also their electromagnetic properties can be affected in an important manner[5, 6, 7, 8, 9]. In Ref. [6] the general expressions for the electromagnetic form factors of a neutrino in an electron gas were calculated and, as an specific application, the correction to the index of refraction of a single neutrino in the presence of an external magnetic field was determined. In this paper we extend the study of Ref. [6] by considering the combined effects of neutrino mixing and the external magnetic field. We derive the equation governing the flavor evolution under such conditions and examine the possible effects of strong magnetic fields on the neutrino oscillations induced by their effective electromagnetic interaction.

It is worth stressing the following. The possibility that neutrinos may have electromagnetic dipole moment interactions which change left-handed neutrinos into right-handed ones can have important consequences in the context of the solar-neutrino puzzle[10], and the combined effect of matter and magnetic fields on neutrino flavor oscillations and spin precession has been studied[11]. However, it is well known that the values of the neutrino magnetic moments that are estimated in the Standard Model fall well below the values for which their effect can be appreciable. While there are schemes in which the values of the neutrino magnetic moments lie in the relevant range, they require ingredients that are not contained in the standard Electroweak Theory. On the contrary, the effect we are considering is quite different. The neutrino electromagnetic form factors calculated in Ref. [6] preserve chirality and are present even in the Standard Model with massless left-handed neutrinos. Those induced terms arise because of the interactions of neutrinos with the particles in the background. In the presence of an external magnetic field the induced form factors, instead of producing spin flip transitions, contribute to the index of refraction and modify the resonance condition for neutrino oscillations in matter.

For simplicity we restrict ourselves to the case of mixing between only two generations, but the same approach can be applied to more general cases. Our starting point is the self-energy of the neutrinos, with momentum k^μ , in the presence of a uniform magnetic field. As shown in Ref. [6], it can

be written as

$$\Sigma_{eff} = (a\not{k} + b\not{u} + c\not{B}_{ext})L, \quad (1)$$

where u^μ is the velocity four-vector of the medium, $L = (1 - \gamma_5)/2$, and $B_{ext}^\mu = 1/2\epsilon^{\mu\nu\alpha\beta}u_\nu F_{\alpha\beta}$, where F is the electromagnetic field tensor. In what follows we work in the rest frame of the background, where $u^\mu = (1, \vec{0})$, $k^\mu = (\omega, \vec{k})$, and $B_{ext}^\mu = (0, \vec{B})$ with \vec{B} being the external magnetic field.

The coefficients a , b , and c are matrices in the neutrino internal space and, in a normal matter gas, composed of electrons, nucleons and their antiparticles, they are given by

$$\begin{aligned} a &= 0, \\ b &= \sqrt{2}G_F Q_Z + \begin{pmatrix} b_e & 0 \\ 0 & 0 \end{pmatrix} \\ c &= \sqrt{2}G_F Q'_Z + \begin{pmatrix} c_e & 0 \\ 0 & 0 \end{pmatrix} \end{aligned} \quad (2)$$

where, to order G_F ,

$$\begin{aligned} b_e &= \sqrt{2}G_F(n_e - n_{\bar{e}}) \\ c_e &= 2\sqrt{2}eG_F \int \frac{d^3p}{(2\pi)^3} \frac{d}{2E} (f_- - f_+), \end{aligned} \quad (3)$$

with e being the electron charge ($e < 0$).¹ We have introduced the electron and positron distributions

$$f_\mp(p) = \frac{1}{e^{\beta(E(p) \mp \mu)} + 1}, \quad (4)$$

and the corresponding number densities

$$n_{e,\bar{e}} = 2 \int \frac{d^3p}{(2\pi)^3} f_\mp(p). \quad (5)$$

In Eq. (2) we have denoted by Q_Z and Q'_Z the contributions arising from the Z -diagram, which are the same for all flavors and, in a normal matter background, are irrelevant for oscillations. However, in environments like the

¹We take the opportunity to stress that in Ref. [6] the symbol e stands everywhere for the electron charge and not for its magnitude.



Figure 1: Diagram (a) gives the dominant contribution (of order $1/m_W^2$) to the neutrino self-energy term of order B^2 , arising from the W exchange interaction. In the local limit of the W propagator, this diagram is equivalent to Diagram (b).

early universe or the core of a supernova, where the neutrinos represent an appreciable fraction of the total density, the neutral-current contributions to the potential energy arising from the ν - ν scattering are not in general proportional to the unit matrix and should be included in the analysis of the resonant flavor transformations[12]. Several approaches exist to describe the neutrino oscillations under such conditions, including a Boltzmann-type kinetic approach[13] based on a density matrix formalism[14], and a treatment based on the FTFT methods[15]. While we are aware that the extra contributions from the ν - ν background interactions must in general be taken into account in a careful numerical study, in what follows we will not consider them further since they can be added at any stage. Of course, they will be important in the particular context of the supernova, for example[16].

In general, the coefficients a , b and c are functions of B . The calculation of Ref. [6] corresponds to retaining only the contribution to the coefficients that is independent of B . Since we are envisaging a situation where cB could be comparable to b , it is pertinent to ask whether the second and higher order terms in B are important or not. In order to answer that, consider the contribution to the neutrino self-energy arising from the diagram in which two external B lines are attached to the electron line of the W -exchange loop diagram, as shown in Fig. 1.

That gives is an additional contribution to b_e such that, instead of the

formula for b_e given in Eq. (3), the result is

$$b_e = \sqrt{2}G_F \left(2 \int \frac{d^3p}{(2\pi)^3} (f_- - f_+) + b_2 B^2 \right), \quad (6)$$

where b_2 is a coefficient independent of B . The crucial point now is to note that the term involving the distribution functions f_{\mp} cannot be identified with the total densities as in Eq. (5), because the latter quantities must now be determined to order B^2 also. The diagram for the B^2 contribution to the electron current density $\langle \bar{e} \gamma^\mu e \rangle$, which can also be written as the trace of $S_F \gamma^\mu$ where S_F is the electron propagator, is identical to Diagram (b) but with the external neutrino lines removed. Thus, by simple inspection of the two diagrams it is easy to recognize that the net number density is given by precisely the same factor that appears in Eq. (6),

$$n_- - n_+ = 2 \int \frac{d^3p}{(2\pi)^3} (f_- - f_+) + b_2 B^2. \quad (7)$$

This is now the relation that determines the chemical potential in terms of the net number density, which is the conserved quantity. If we proceed to eliminate the chemical potential in favor of $n_- - n_+$, the result is that the formula for b_e reduces again to that in Eq. (3), while c_e acquires a dependence on B through the implicit dependence of the f_{\mp} on B , as a consequence of solving for the chemical potential in Eq. (7). The same argument applies to higher order terms in B . This can be seen easily by considering, for example, diagrams with more B lines attached to the electron, and noting that they are equivalent to the electron loop of Diagram (a) but with the additional external B lines. All this amounts to the statement that, to this order in G_F , the neutrino self-energy is given by just $\langle \bar{e} \gamma^\mu e \rangle \gamma_\mu L$. Then, according to Eq. (1), b_e is determined by $\langle e^\dagger e \rangle$ which is just the net number density. Thus, to summarize, the formula for b_e in Eq. (3) is, to order G_F , independent of B . On the other hand, by adopting Eq. (5) as the relation between the chemical potential and the net number density, we are neglecting the dependence of c_e on the magnetic field².

The coefficients b, c in Eq. (2) are written in the flavor basis. In this basis, the equation that determines the dispersion relation and wave functions of

²For the purpose of studying this dependence, a better procedure is probably to start from the expression for the neutrino self-energy in terms of the electron propagator in an external magnetic field, as recently carried out by Elmfors, Grasso and Raffelt[17]

the propagating modes is

$$(\not{k} - m - \Sigma_{eff})\psi = 0, \quad (8)$$

where,

$$m = U \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} U^\dagger, \quad (9)$$

with U being the matrix that relates the fields of definite mass ν_{Li} to the flavor fields $\nu_{L\alpha} = \sum_i U_{\alpha i} \nu_{Li}$ ($\alpha = e, \mu$). In the Weyl representation

$$\psi_R = \begin{pmatrix} \xi \\ 0 \end{pmatrix}, \quad \psi_L = \begin{pmatrix} 0 \\ \eta \end{pmatrix}, \quad (10)$$

and Eq. (8) becomes the set of coupled equations

$$\begin{aligned} (\omega - b + \vec{\sigma} \cdot \vec{\kappa} - c\vec{\sigma} \cdot \vec{B})\eta - m\xi &= 0, \\ (\omega - \vec{\sigma} \cdot \vec{\kappa})\xi - m\eta &= 0. \end{aligned} \quad (11)$$

Using the second of these to eliminate ξ from the first one, yields the following equation for η

$$\left[(\omega - b + \vec{\sigma} \cdot \vec{\kappa} - c\vec{\sigma} \cdot \vec{B}) - (\omega + \vec{\sigma} \cdot \vec{\kappa}) \frac{m^2}{\omega^2 - \kappa^2} \right] \eta = 0, \quad (12)$$

while ξ is then determined as

$$\xi = (\omega + \vec{\sigma} \cdot \vec{\kappa}) \frac{m}{\omega^2 - \kappa^2} \eta. \quad (13)$$

We are interested in the solutions of Eq. (12) with positive energy, corresponding to the (neutrinos) particle solutions. In the absence of the magnetic field they correspond to negative helicity spinors of the form

$$\eta = e_{1,2} \phi_-, \quad (14)$$

where ϕ_λ is the Pauli spinor with definite helicity, that satisfies

$$(\vec{\sigma} \cdot \hat{\kappa}) \phi_\lambda = \lambda \phi_\lambda \quad (\lambda = \pm). \quad (15)$$

The $e_{1,2}$ are vectors in flavor space, determined by solving the eigenvalue problem

$$H e_i = \omega_i e_i, \quad (16)$$

where

$$H = \kappa + \frac{m^2}{2\kappa} + b. \quad (17)$$

The expressions for $\omega_{1,2}$ and $e_{1,2}$ are given explicitly in Eqs. (A.10) and (A.14) of Ref. [15]. To arrive at Eq. (17), the substitution

$$(\omega + \vec{\sigma} \cdot \vec{\kappa}) \frac{m^2}{\omega^2 - \kappa^2} \eta = \frac{m^2}{\omega + \kappa} \eta \simeq \frac{m^2}{2\kappa} \eta \quad (18)$$

has been made.

In the presence of the magnetic field, the solutions to Eq. (12) do not correspond to purely negative helicity spinors any longer because the matrices $\vec{\sigma} \cdot \vec{B}$ and $\vec{\sigma} \cdot \vec{\kappa}$ generally do not commute. Therefore, the solution must be sought in the form

$$\eta = x\phi_- + x'\phi_+, \quad (19)$$

where x, x' are two-component vectors in flavor space. Substituting Eq. (19) into Eq. (12) we obtain two coupled equations for x and x' , and from them is easy to verify that $x' \sim (c\vec{B} \cdot \hat{\kappa}/\kappa)x$. Then, retaining terms that are at most linear in the small quantities $b, c, m^2/2\kappa$ in the equation for x , the dispersion relations $\omega_{1,2}$ and the corresponding vectors $x = e_{1,2}$ are obtained by solving Eq. (16), but with the Hamiltonian

$$H_B = \kappa + b - c\vec{B} \cdot \hat{\kappa} + \frac{m^2}{2\kappa} \quad (20)$$

in place of H .

Imitating the arguments given in Ref. [15], we then arrive at the following picture: The Dirac wave function for a relativistic left-handed neutrino with momentum $\vec{\kappa}$ propagating in matter in the presence of an external magnetic field is, in the Weyl representation,

$$\psi_L = e^{i\vec{\kappa} \cdot \vec{x}} \begin{pmatrix} 0 \\ \phi_- \end{pmatrix} \chi(t), \quad (21)$$

where, in a homogenous medium, $\chi(t) = \sum_{i=1,2} (e_i^\dagger \chi(0)) e_i e^{-i\omega_i t}$. For an inhomogenous medium, the flavor-space spinor $\chi(t)$ is the solution of

$$i \frac{d\chi}{dt} = H_B \chi, \quad (22)$$

which is the extension of the MSW equation to the situation we are considering. The components $\chi_{e,\mu}$ of $\chi(t)$ give the amplitude to find the neutrino in the corresponding state of definite flavor, at a distance $r \simeq t$ from the production point ($t_0 = 0$). In Eq. (21) we discarded the right-handed component ψ_R of the Dirac spinor since it is of order $m/2\kappa$ as compared to the left-handed component, and we also neglect the positive-helicity component $x'\phi_+$ since it is of order $\frac{cB}{\kappa}$ relative to $x\phi_-$.

We now consider the possible relevance of the effect of the extra terms due to the magnetic field on the resonant flavor conversion. At each point along the neutrino path, H_B can be diagonalized by the unitary transformation

$$U_m(r) = \begin{pmatrix} \cos \theta_m & \sin \theta_m \\ -\sin \theta_m & \cos \theta_m \end{pmatrix}, \quad (23)$$

with the mixing angle in matter θ_m determined by

$$\sin 2\theta_m = \frac{\Delta_0 \sin 2\theta}{\sqrt{(\Delta_0 \cos 2\theta - v)^2 + \Delta_0^2 \sin^2 2\theta}}, \quad (24)$$

where

$$v(r) = b_e(r) - c_e(r)\vec{B} \cdot \hat{\kappa}, \quad (25)$$

and $\Delta_0 \equiv (m_2^2 - m_1^2)/2\kappa$. The denominator in Eq. (24) corresponds to the difference between the instantaneous energy eigenvalues of the neutrino modes $\omega_2 - \omega_1$. For antineutrinos the b_e and c_e terms change sign and

$$\sin 2\bar{\theta}_m = \frac{\Delta_0 \sin 2\theta}{\sqrt{(\Delta_0 \cos 2\theta + v)^2 + \Delta_0^2 \sin^2 2\theta}}. \quad (26)$$

According to the above formulas, the mixing angle in matter is modified by the neutrino (antineutrino) interaction with the magnetic field. As functions of v , $\sin^2 2\theta_m$ and $\sin^2 2\bar{\theta}_m$ exhibit the characteristic form of a Breit-Wigner resonance. For the neutrinos the resonance condition ($\sin 2\theta_m = 1$) is

$$v(r_R) = \Delta_0 \cos 2\theta, \quad (27)$$

while for the antineutrinos we have

$$v(\bar{r}_R) = -\Delta_0 \cos 2\theta. \quad (28)$$

The width of the resonance σ is given by the length of the interval of values of $v(r)$, centered around $v(r_R)$, such that $\sin^2 2\theta_m \geq \frac{1}{2}$, and similarly for $\sin^2 2\bar{\theta}_m$. In both cases,

$$\sigma = 2\Delta_0 \sin 2\theta. \quad (29)$$

In principle, Eqs. (27) and (28) can be verified simultaneously in a medium with $B \neq 0$. This is a novel feature that contrasts with the situation without the magnetic field, where only neutrinos, but not antineutrinos, can go through a resonant region. For negligible values of $\Delta_0 \cos 2\theta$ both conditions reduce to

$$v(r_R) \simeq v(\bar{r}_R) \simeq 0, \quad (30)$$

and in such case the resonance for neutrinos and antineutrinos occur within regions which are close. More precisely, the above approximations will be good to the extent the differences $|r_R - \tilde{r}|$ and $|\bar{r}_R - \tilde{r}|$, where $v(\tilde{r}) = 0$, are small compared with r_R itself. In turn, this implies that

$$\lambda_R \ll r_R, \quad (31)$$

where

$$\lambda_R \equiv \left| \frac{1}{v} \frac{dv}{dr} \right|_{r=r_R}^{-1} \quad (32)$$

is the characteristic scale length of v at the neutrino resonance, and we have taken into account the fact that $\lambda_R \simeq \frac{1}{2}|r_R - \bar{r}_R|$. It is pertinent to remark that the condition given by Eq. (31) does not necessarily imply that the resonance regions of ν and $\bar{\nu}$ overlap. This will happen whenever the separation between the two resonance points is smaller than the average spatial extension of the regions. For the neutrinos, the actual spatial extension of the resonance region is given by $\delta_\nu = \sigma/|v'(r_R)| \simeq 2\lambda_R \tan 2\theta$, where $v' = dv/dr$, with an analogous expression for $\delta_{\bar{\nu}}$. The condition that both regions overlap is $|r_R - \bar{r}_R| < \frac{1}{2}(\delta_\nu + \delta_{\bar{\nu}})$, and assuming $v'(r_R) \simeq v'(\bar{r}_R)$ it simplifies to $\tan 2\theta \gtrsim 1$, which requires large values of the mixing angle.

If the magnetic fields are sufficiently strong that Eq. (30) can be satisfied, then the phenomenon of resonant oscillations can take place, without a severe requirement on the masses and mixing angles of the neutrinos. Thus, even if the values of these parameters are constrained by the condition for resonant oscillations in the Sun and/or other physical phenomena, the supernova may simultaneously support resonant oscillations under the conditions just stated. In order to estimate the order of magnitude of the magnetic field needed to

satisfy Eq. (30), in what follows we evaluate c_e in various limiting cases, which are easily obtained from Eq. (3).

Degenerate gas. In the limit $T \rightarrow 0$, we have $n_{\bar{e}} = 0$ and

$$c_e = -\sqrt{2}G_F m_e \mu_B \left(\frac{3n_e}{\pi^4} \right)^{1/3}. \quad (33)$$

where $\mu_B = e/2m_e$ is the Bohr magneton. Eq. (30) then requires

$$-\mu_B \vec{B} \cdot \hat{\kappa} = \frac{n_e^{2/3}}{m_e} \left(\frac{\pi^4}{3} \right)^{1/3}, \quad (34)$$

which can be written in the form

$$\left(\frac{\vec{B} \cdot \hat{\kappa}}{10^{14} \text{gauss}} \right) = 67 \left(\frac{Y_e}{0.3} \right)^{2/3} \left(\frac{\rho}{\rho_0} \right)^{2/3}, \quad (35)$$

where ρ is the mass density, Y_e is the fractional number density of electrons and $\rho \equiv 10^{10} \text{g/cm}^3$.

Ultrarelativistic non-degenerate gas. In the ultrarelativistic limit, we obtain

$$c_e = -\frac{G_F e}{\sqrt{2}\pi^2} \mu. \quad (36)$$

Approximating the Fermi-Dirac distribution by its classical limit we obtain

$$\frac{\pi^2 \beta^3}{4} (n_e - n_{\bar{e}}) = \sinh \beta \mu. \quad (37)$$

Then, for small values of $n_e - n_{\bar{e}}$, Eq. (36) reduces to

$$c_e \simeq -\frac{G_F}{2\sqrt{2}} m_e \mu_B \beta^2 (n_e - n_{\bar{e}}). \quad (38)$$

and the condition in Eq. (30) translates to

$$\left(\frac{\vec{B} \cdot \hat{\kappa}}{10^{14} \text{gauss}} \right) = (1.4 \times 10^3) \left(\frac{T}{10 \text{MeV}} \right)^2. \quad (39)$$

Non-relativistic Boltzmann gas. For this case we can borrow the result given in Ref. [6],

$$c_e = -\sqrt{2}G_F\mu_B\beta(n_e - n_{\bar{e}}), \quad (40)$$

and Eq. (30) then becomes

$$\left(\frac{\vec{B} \cdot \hat{k}}{10^{14}gauss}\right) = 17 \left(\frac{T}{10MeV}\right). \quad (41)$$

It is noteworthy that, in the classical limit, the resonance condition becomes approximately independent of the number densities.

The estimates given in Eqs. (35) and (41) indicate that the phenomenon we have considered may be relevant in the study of resonant oscillations in the supernova, where the densities are typically of order $10^{10}g/cm^3$, and magnetic fields of order $10^{14}gauss$ have been considered. Our results suggest that the effect is worthy of further attention and detailed numerical studies in order to asses the range of implications in a concrete way.

The possibility that the induced neutrino electromagnetic vertex may contribute to the index of refraction in the presence of an external magnetic field has also been recently pointed out[18]. However, that work did not include the presence of the matter term (b_e) in the evolution equation, and therefore did not consider the implications of the combined effect of that term plus the magnetic field term (c_e) on the resonant oscillations.

After this work was completed and while this manuscript was being prepared, we received a preprint[19] discussing the same effect that we consider here. However, the physical outlook and the conclusions reached there regarding the new resonant condition and its implications are different from ours.

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